THE GEOMETRY OF MATROIDS
LECTURE 3 EXERCISES

1. **Circuits in graphic and representable matroids**
   (a) Let G be a finite graph. Describe the circuits of $M(G)$.
   (b) Let $\mathcal{A}$ be a vector configuration. Describe the circuits of $M(\mathcal{A})$.

2. **Matroids with few circuits**
   (a) Give an example of a matroid which has no circuits. Can you describe all such matroids?
   (b) Give an example of a matroid which has exactly one circuit. Can you describe all such matroids?
   (c) Give an example of a matroid which has exactly two circuits. Can you describe all such matroids?

3. **Paving matroids**
   A matroid M is called a **paving matroid** if $|C| \geq \mathrm{rk}(M)$ for every circuit $C \in \mathcal{C}(M)$.
   (a) Show that if C is a circuit of *any* matroid M, then $|C| \leq \mathrm{rk}(M) + 1$.
   (b) Let G be a connected graph. Under what conditions is $M(G)$ a paving matroid?
   (c) Let $\mathcal{A}$ be a vector configuration spanning a 3-dimensional vector space V. Under what conditions is $M(\mathcal{A})$ a paving matroid?
   (d) How does your answer to part (c) change when we increase the dimension of the vector space?