THE GEOMETRY OF MATROIDS
LECTURE 14 EXERCISES

1. **The lattice of flats of some matroids**
   For each of the matroids below, draw the Hasse diagram of the lattice of flats.
   (a) $U_{0,4}$
   (b) $U_{1,4}$
   (c) $U_{2,4}$
   (d) $U_{3,4}$
   (e) $U_{4,4}$
   (f) F_7
   (g) F_7^-
   (h) $M(K_4)$

2. **The poset dual of a lattice**
   Let $\mathcal{P}$ be a poset. Let $\mathcal{P}'$ be the poset defined by $x \leq y$ in $\mathcal{P}'$ if and only if $x \geq y$ in $\mathcal{P}$. In other words, the Hasse diagram for $\mathcal{P}'$ is obtained by flipping the Hasse diagram for $\mathcal{P}$ upside-down.
   (a) Prove that if $\mathcal{P}$ is a lattice, then so is $\mathcal{P}'$.
   (b) Show, by examples, that if $\mathcal{P}$ is a geometric lattice, then $\mathcal{P}'$ may or may not be geometric.

3. **The lattice of flats of a relaxation**
   Let H be a circuit-hyperplane of a matroid M, and let M' be the matroid obtained from M by relaxing H (cf. Lecture 7 Exercise 3). Describe the lattice of flats of M' in terms of the lattice of flats of M.