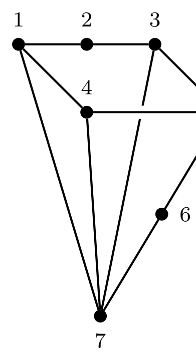
THE GEOMETRY OF MATROIDS
LECTURE 17 EXERCISES

1. **Geometric representation of the contraction**
   Let M be the matroid on $[7] = \{1, \ldots, 7\}$ with geometric representation shown below.



   Draw geometric representations of the matroids $M/1$, $M/4$, $M/7$, and $M/\{1, 2, 3\}$.

2. **Loops in the contraction**
   Let M be a matroid on E and let $T \subseteq E$. Prove that the contraction M/T has no loops if and only if T is a flat.

3. **Circuits in the contraction**
   Let M be a matroid on E and let $T \subseteq E$.
   (a) Show, by giving examples, that any of the following possibilities may occur when $C \in \mathcal{C}(M)$ is a circuit of M.
      (i) $C \setminus T$ is a circuit of the contraction M/T.
      (ii) $C \setminus T$ is empty.
      (iii) $C \setminus T$ is non-empty and is not a circuit in M/T.
   (b) Prove that the circuits of $\mathcal{C}(M/T)$ are the minimal non-empty sets in the collection

$$\{C \setminus T \mid C \in \mathcal{C}(M)\}.$$

4. $\star$**Deletion vs. contraction**
   Let M be a matroid on E and let $T \subseteq E$.
   (a) Show that $\mathcal{I}(M/T) \subseteq \mathcal{I}(M \setminus T)$. That is, if $I \subseteq E \setminus T$ is independent in M/T, then I is independent in $M \setminus T$.
   (b) Show, by example, that the containment in part (a) may be strict.
   (c) Prove that $M/T = M \setminus T$ if and only if $\mathrm{rk}_M(T) + \mathrm{rk}_M(E \setminus T) = \mathrm{rk}(M)$.
   (d) Conclude that $M/e = M \setminus e$ if and only if e is a loop or a coloop of M.