THE GEOMETRY OF MATROIDS
LECTURE 18 EXERCISES

1. **Minors of a direct sum**
   Let M_1 and M_2 be matroids on disjoint ground sets E_1 and E_2, respectively. Let $T_1 \subseteq E_1$, $T_2 \subseteq E_2$, and $T = T_1 \cup T_2$.
   (a) Show that
   $$(M_1 \oplus M_2) \setminus T = (M_1 \setminus T_1) \oplus (M_2 \setminus T_2).$$
   (b) Show that
   $$(M_1 \oplus M_2)/T = (M_1/T_1) \oplus (M_2/T_2).$$

2. **The dual of a direct sum**
   Let M_1 and M_2 be matroids on disjoint ground sets. Prove that
   $$(M_1 \oplus M_2)^* = M_1^* \oplus M_2^*.$$

3. $\star$**An excluded minor characterization of paving matroids**
   Recall that a matroid M is a **paving matroid** if $|C| \geq \mathrm{rk}(M)$ for every circuit $C \in \mathcal{C}(M)$.
   (a) Show that paving matroids form a minor-closed class. That is, every minor of a paving matroid is paving.
   (b) Show that $U_{2,2} \oplus U_{0,1}$ is the unique excluded minor for the class of paving matroids. That is, a matroid is paving if and only if it has no minor isomorphic to $U_{2,2} \oplus U_{0,1}$.