

**THE GEOMETRY OF MATROIDS**  
**LECTURE 22 EXERCISES**

1. **★Log-concavity**

A finite sequence  $(a_0, a_1, \dots, a_n)$  of positive numbers is **log-concave** if  $a_i^2 \geq a_{i-1}a_{i+1}$  for all  $1 \leq i \leq n-1$ .

(a) Show that if  $(a_i)$  is a log-concave sequence, then

$$a_{i-1}a_i \geq a_{i-2}a_{i+1}$$

for all  $2 \leq i \leq n-1$ .

(b) Define  $b_0 = a_0$  and  $b_i = a_i + a_{i-1}$  for  $1 \leq i \leq n$ . Prove that if  $(a_i)$  is log-concave, then  $(b_i)$  is also log-concave.

2. **Strong log-concavity**

A sequence  $(a_i)$  is **strongly log-concave** if

$$a_j a_k \geq a_i a_l$$

for all  $i \leq j \leq k \leq l$  such that  $i + l = j + k$ .

Prove that  $(a_i)$  is log-concave if and only if it is strongly log-concave.