Flats of $M(G)$

G finite graph $\rightsquigarrow$ $M(G)$ with rank function

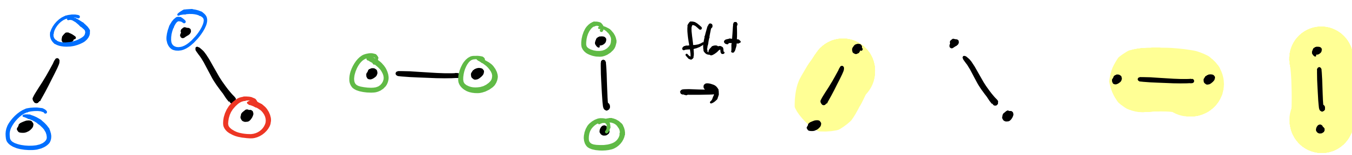
$$\mathrm{rk}_{M(G)}(X) = \#V(G) - \#\{\text{conn. comp. of } G[X]\}$$

for $X \subseteq E(G)$.

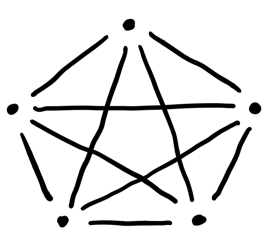
Then X is a flat $\iff$ any edge in $E(G) \setminus X$ connects two components of $G[X]$.

$\iff$ there is a partition $\{V_1, \dots, V_k\}$ of $V(G)$ such that

$$X = E(G[V_1]) \sqcup E(G[V_2]) \sqcup \cdots \sqcup E(G[V_k])$$

$G =$ 

Ex: $G = K_5$



$\mathrm{rk}(M(K_5)) = 4$

r	flats of rank r	#flats of rank r
4	K_5	1
3	5 K_4 subgraphs 10 $K_3 \sqcup K_2$ subgraphs	15
2	15 pairs of disjoint edges 10 K_3 subgraphs	25
1	any single edge	10
0	$\varnothing$	1

Def: The <u>uniform matroid</u> of rank r on n elements $(0 \le r \le n)$ is the matroid $U_{r,n}$ with ground set

$$E(U_{r,n}) = [n] = \{1, \ldots, n\}$$

and independent set

$$\mathcal{I}(U_{r,n}) = \{ I \subseteq [n] \mid |I| \le r \}.$$

- <u>Check:</u> $rk(U_{r,n}) = r$
- $U_{n,n}$ is called a <u>free matroid</u> or a <u>boolean matroid</u>.
- $U_{0,0}$ is called the <u>empty matroid</u>. It is the unique matroid on $\emptyset$.

<u>Questions:</u>
- What are the bases, circuits, rank function, closure op., flats?
- Is $U_{r,n}$ graphic? Representable?