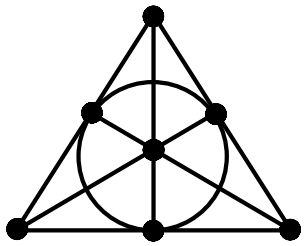
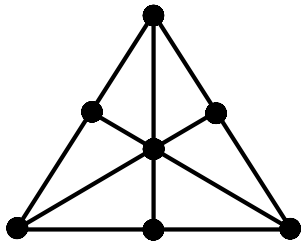
Fano vs. non-Fano



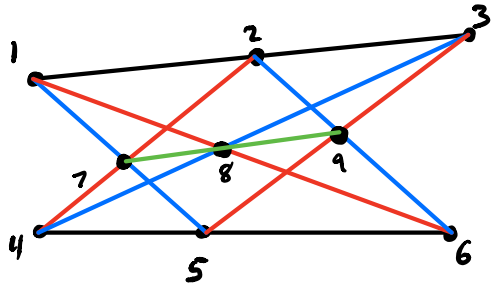
F_7



F_7^-

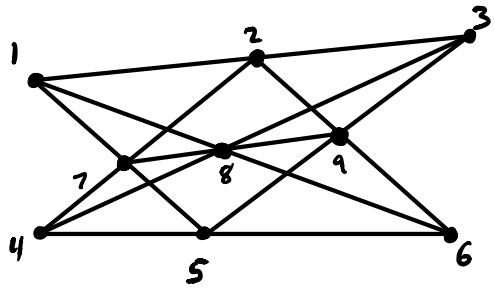
Exercise 3 (last time): F_7^- is a relaxation of F_7

Thm (Pappus, ~320 AD) If 9 points are arranged in the following configuration (in proj. plane over any field)



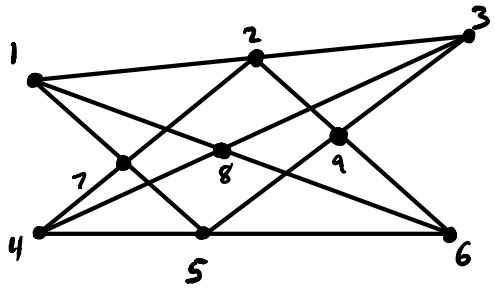
then 789 are collinear.

Cor: The Pappus matroid, the matroid of the configuration



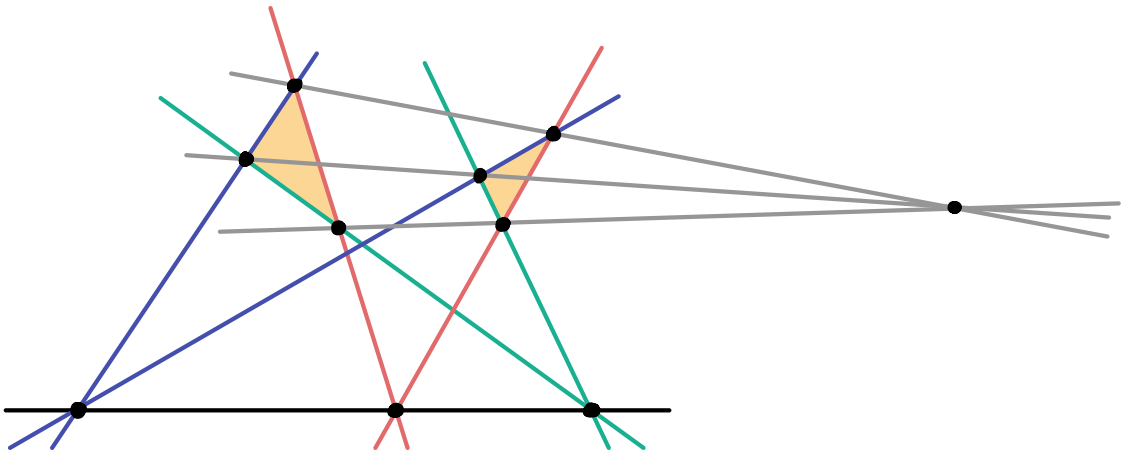
is representable over every field except $\mathbb{F}_2$, $\mathbb{F}_3$, and $\mathbb{F}_5$.

Cor: The **non-Pappus matroid**, the matroid of the configuration



is not representable over any field.

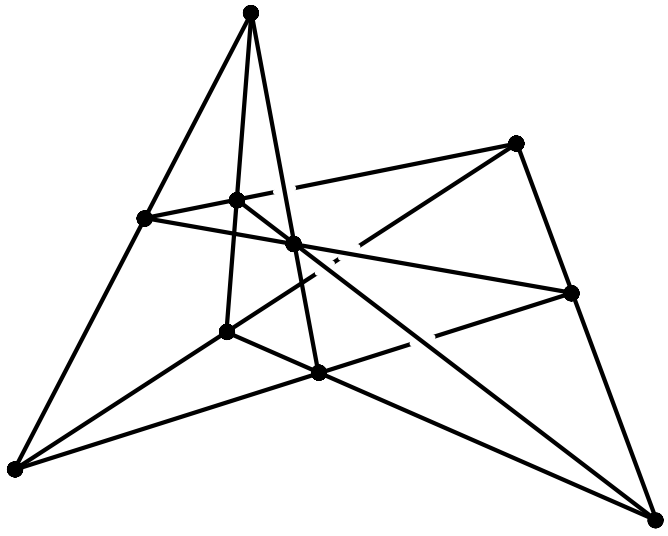
Ex: Desargues configuration:



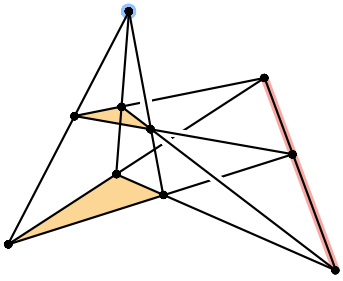
Desargues Thm: 2 triangles are in perspective axially if and only if they're in perspective centrally

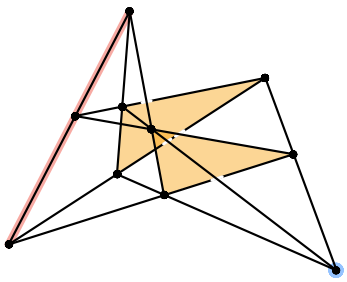
- True in any projective space over a division ring.
- Pappus $\Rightarrow$ Desargues
- A "projective geometry" is $\mathbb{P}^n_K$ for K a field $\Leftrightarrow$ Pappus
  is $\mathbb{P}^n_D$ for D a div. ring $\Leftrightarrow$ Desargues

Ex: The 3-dim Desargues configuration is more symmetric



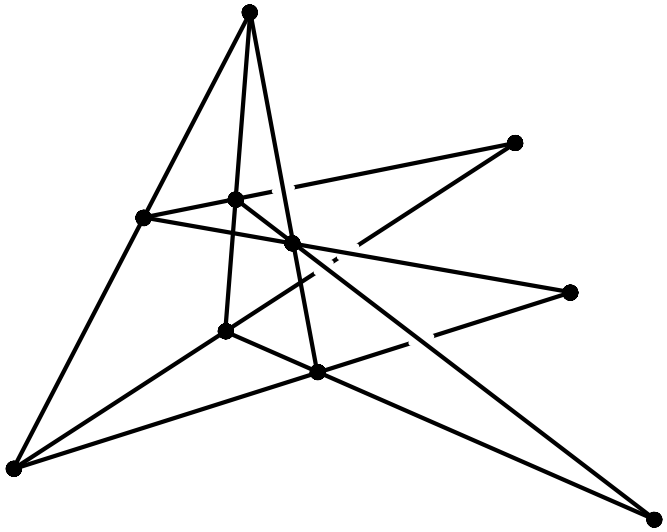
Any of the 10 points can be the center of perspectivity.





- Realized by 5 planes in generic position
  - $\binom{5}{2}$ double intersections = 10 3-pointed lines
  - $\binom{5}{3}$ triple intersections = 10 points
- Each point is in 3 3-pointed lines (and 3 2-pointed lines)
- Each plane contains 6 points
- It's a geometric rep. of $M(K_5)$!

Ex: The non-Desargues matroid



is not representable over any field.