

## Some other constructions

- The Higgs lift of  $M$  is

$$\text{lift}(M) = (\text{trunc}(M^*))^*$$

"generically increase the rank by 1"

- The free extension of  $M$  by an element  $e \in E$  is

$$M + e = \text{trunc}(M \oplus U_{1,1})$$

"add a point in generic position"

$$(M + e) \setminus e = M$$

$$(M + e) / e = \text{trunc}(M)$$

- The free coextension of  $M$  by  $e \in E$  is

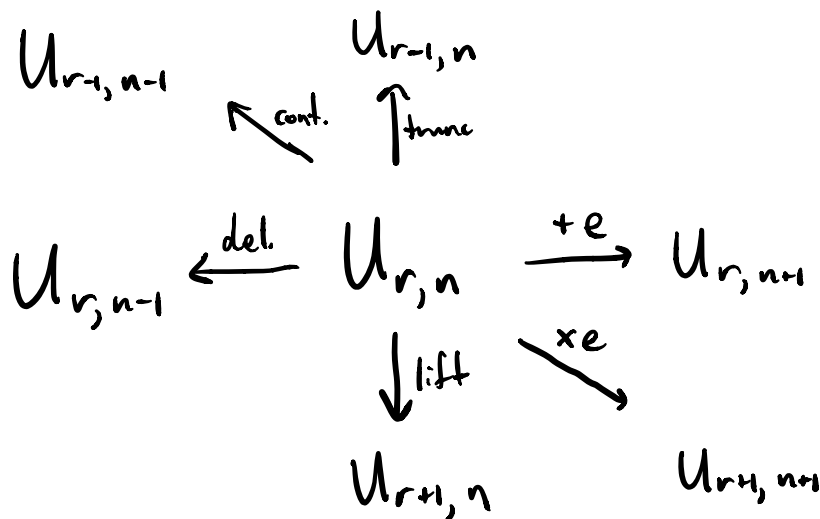
$$M \times e = (M^* + e)^* = \text{lift}(M \oplus U_{0,1})$$

???

$$(M \times e) / e = M$$

$$(M \times e) \setminus e = \text{lift}(M)$$

Ex: If  $1 \leq r \leq n-1$ ,



## The Characteristic Polynomial

Def: The characteristic polynomial of a matroid  $M$  on  $E$  is

$$\chi_M(q) = \sum_{S \subseteq E} (-1)^{|S|} q^{\text{crk}(S)} \in \mathbb{Z}[q].$$

Ex:  $\chi_{U_{0,0}}(q) = (-1)^{|\emptyset|} q^{\text{crk}(\emptyset)} = 1.$

Ex:  $M = U_{3,4}$

If  $|S| \leq 3$ , then  $\text{crk}(S) = 3 - \text{rk}(S)$   
 $= 3 - |S|$

If  $|S| = 4$ , then  $\text{crk}(S) = 3 - 3 = 0$ .

So

$$\chi_{U_{3,4}}(q) = \underset{\substack{\uparrow \\ |S|=0}}{q^3} - \underset{\substack{\uparrow \\ |S|=1}}{4q^2} + \underset{\substack{\uparrow \\ |S|=2}}{6q} - \underset{\substack{\uparrow \\ |S|=3}}{4} + \underset{\substack{\uparrow \\ |S|=4}}{1}$$

$$= q^3 - 4q^2 + 6q - 3$$