Some other constructions

- The <u>Higgs lift</u> of M is

$$\mathrm{lift}(M) = (\mathrm{trunc}(M^*))^*$$

"generically increase the rank by 1"

- The <u>free extension</u> of M by an element $e \notin E$ is

$$M + e = \mathrm{trunc}(M \oplus U_{1,1})$$

"add a point in generic position"

$$(M+e) \setminus e = M$$

$$(M+e) / e = \mathrm{trunc}(M)$$

- The <u>free coextension</u> of M by $e \notin E$ is

???

$$M \times e = (M^* + e)^* = \mathrm{lift}(M \oplus U_{0,1})$$

$$(M \times e)/e = M$$

$$(M \times e) \setminus e = \mathrm{lift}(M)$$

$\underline{\text{Ex}}$: If $1 \le r \le n-1$,

$$\begin{tikzcd}
U_{r-1,n-1} & & U_{r-1,n} & \\
& & & \\
U_{r,n-1} & & U_{r,n} \arrow[ll, "\text{del.}"'] \arrow[r, "+e"] \arrow[u, "\text{trunc}"'] \arrow[ull, "\text{cont.}"'] \arrow[d, "\text{lift}"] \arrow[dr, "xe"] & U_{r,n+1} \\
& & U_{r+1,n} & U_{r+1,n+1}
\end{tikzcd}$$

The Characteristic Polynomial

$\underline{\text{Def}}$: The <u>characteristic polynomial</u> of a matroid M on E is

$$\chi_M(q) = \sum_{S \subseteq E} (-1)^{|S|} q^{\mathrm{crk}(S)} \in \mathbb{Z}[q].$$

$\underline{\text{Ex}}$: $\chi_{U_{0,0}}(q) = (-1)^{|\emptyset|} q^{\mathrm{crk}(\emptyset)} = 1$.

Ex: $M = U_{3,4}$

If $|S| \le 3$, then $\operatorname{crk}(S) = 3 - \operatorname{rk}(S)$
$= 3 - |S|$

If $|S| = 4$, then $\operatorname{crk}(S) = 3 - 3 = 0.$

So

$$\chi_{U_{3,4}}(q) = \underset{|S|=0}{q^3} - \underset{|S|=1}{4q^2} + \underset{|S|=2}{6q} - \underset{|S|=3}{4} + \underset{|S|=4}{1}$$

$$= q^3 - 4q^2 + 6q - 3$$