

Problem A; Abbott: 3.4.5, 3.4.7

**A**

- (a) Find an example of a disconnected set whose closure is connected.
- (b) If  $A$  is connected, is  $\bar{A}$  necessarily connected?

**3.4.5** Let  $A$  and  $B$  be nonempty subsets of  $\mathbb{R}$ . Show that if there exist disjoint open sets  $U$  and  $V$  with  $A \subseteq U$  and  $B \subseteq V$ , then  $A$  and  $B$  are separated.

**3.4.7** A set  $E$  is *totally disconnected* if, given any two distinct points  $x, y \in E$ , there exist separated sets  $A$  and  $B$  with  $x \in A, y \in B$ , and  $E = A \cup B$ .

(a) Show that  $\mathbb{Q}$  is totally disconnected.

(b) Is the set of irrational numbers totally disconnected?