Math 315
Homework #13
5/23/2017

Abbott: 4.3.1, 4.3.9, 4.4.3, 4.4.9

4.3.1 Let $g(x) = \sqrt[3]{x}$.

(a) Prove that g is continuous at $c = 0$.

(b) Prove that g is continuous at a point $c \neq 0$. (The identity

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

will be helpful.)

4.3.9 Assume $h\colon \mathbb{R} \to \mathbb{R}$ is continuous on $\mathbb{R}$ and let $K = \{x : h(x) = 0\}$. Show that K is a closed set.

4.4.3 Show that $f(x) = 1/x^2$ is uniformly continuous on the set $[1, \infty)$ but not on the set $(0, 1]$.

4.4.9 (Lipschitz Functions) A function $f: A \to \mathbb{R}$ is called *Lipschitz* if there exists a bound $M > 0$ such that

$$\left| \frac{f(x) - f(y)}{x - y} \right| \leq M$$

for all $x, y \in A$. Geometrically speaking, a function f is Lipschitz if there is a uniform bound on the magnitude of the slopes of lines drawn through any two points on the graph of f.

(a) Show that if $f: A \to \mathbb{R}$ is Lipschitz, then it is uniformly continuous on A.

(b) Is the converse statement true? Are all uniformly continuous functions necessarily Lipschitz?