Abbott: 6.3.1, 6.4.3

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**6.3.1** Consider the sequence of functions defined by

$$g_n(x) = \frac{x^n}{n}.$$

(a) Show $(g_n)$ converges uniformly on $[0,1]$ and find $g = \lim g_n$. Show that $g$ is differentiable and compute $g'(x)$ for all $x \in [0,1]$.

(b) Now, show that $(g_n')$ converges on $[0,1]$. Is the convergence uniform? Set $h = \lim g_n'$ and compare $h$ and $g'$. Are they the same?

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**6.4.3**

(a) Show that
$$g(x) = \sum_{n=0}^{\infty} \frac{\cos(2^n x)}{2^n}$$
is continuous on all of $\mathbb{R}$.

(b) The function $g$ was cited in Section 5.4 as an example of a continuous nowhere differentiable function. What happens if we try to use Theorem 6.4.3 to explore whether $g$ is differentiable?