Math 315
Homework #2
4/11/2017

Abbott: 1.3.5, 1.3.8, 1.4.1, 1.4.3

1.3.5 As in Example 1.3.7, let $A \subseteq \mathbb{R}$ be nonempty and bounded above, and let $c \in \mathbb{R}$. This time, define the set cA by $cA = \{ca : a \in A\}$.

(a) If $c \geq 0$, show that $\sup(cA) = c \sup A$.

(b) Postulate a similar type of statement for $\sup(cA)$ for the case $c < 0$.

1.3.8 Compute, without proofs, the suprema and infima of the following sets:

(a) $\{m/n : m, n \in \mathbb{N} \text{ with } m < n\}$.

(b) $\{(-1)^m/n : m, n \in \mathbb{N}\}$.

(c) $\{n/(3n+1) : n \in \mathbb{N}\}$.

(d) $\{m/(m+n) : m, n \in \mathbb{N}\}$.

1.4.1 Recall that $\mathbb{I}$ stands for the set of irrational numbers.

(a) Show that if $a, b \in \mathbb{Q}$, then ab and $a + b$ are elements of $\mathbb{Q}$ as well.

(b) Show that if $a \in \mathbb{Q}$ and $t \in \mathbb{I}$, then $a + t \in \mathbb{I}$ and $at \in \mathbb{I}$ as long as $a \neq 0$.

(c) Part (a) can be summarized by saying that $\mathbb{Q}$ is closed under addition and multiplication. Is $\mathbb{I}$ closed under addition and multiplication? Given two irrational numbers s and t, what can we say about $s + t$ and st?

1.4.3 Prove that $\bigcap_{n=1}^{\infty}(0, 1/n) = \emptyset$. Notice that this demonstrates that the intervals in the Nested Interval Property must be closed for the conclusion of the theorem to hold.