

Abbott: 2.5.5, 2.5.7

2.5.5 Assume (a_n) is a bounded sequence with the property that every convergent subsequence of (a_n) converges to the same limit $a \in \mathbb{R}$. Show that (a_n) must converge to a .

[*Hint:* Suppose (a_n) does not converge to a . Carefully use the definition of convergence to produce a positive number $\epsilon_0 > 0$ and a subsequence—not necessarily convergent—of (a_n) which never enters the neighborhood $V_{\epsilon_0}(a)$.]

2.5.7 Extend the result proved in Example 2.5.3 to the case $|b| < 1$; that is, show $\lim(b^n) = 0$ if and only if $-1 < b < 1$.