Homework 1

Math 3345 – Spring 2024 – Kutler

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 2 Exercise 1]** Let P and Q be logical sentences. Prove the second DeMorgan Law

   $$\neg(P \vee Q) \text{ is logically equivalent to } \neg P \wedge \neg Q$$

   in two ways:

   (a) By means of a truth table;

   (b) By means of an explanation in words.

2. Let P, Q, and R be logical sentences. Show by means of a truth table that the sentence $P \wedge (Q \vee R)$ is not logically equivalent to the sentence $(P \wedge Q) \vee R$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Explain why the sentence

   "This sentence is false."

   is not a logical sentence.

2. For each sentence below, describe (in interval notation) the set of real numbers x for which the sentence is true.

   (a) $(x > 1) \wedge (x < 3)$

   (b) $(x > 1) \vee (x < 3)$

   (c) $\neg((x > 1) \wedge (x < 3))$

   (d) $\big(\neg\left((x > 1) \wedge (x < 3)\right)\big) \vee ((x < 2) \wedge (x > 1))$.

   For example, the sentence $(x \geq 0) \wedge (x < 5)$ is true for exactly the real numbers x in the half-open interval $[0, 5)$.

   You may have to write your answer as a union of intervals.