Homework 10

Math 3345 – Spring 2024 – Kutler

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 5 Exercise 3(a)(b)]**

   (a) Prove by induction that for each $n \in \mathbb{N}$,

   $$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$$

   (b) Explain why it follows from part (a) and Section 5 Exercise 1 that for each $n \in \mathbb{N}$,

   $$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2.$$

   Note: Section 5 Exercise 1 was our first induction example in class (see the notes from Lecture 9).

2. **[Falkner Section 4 Exercise 14]** Let $a, b, c \in \mathbb{Z}$. Prove the following statements.

   (a) a divides a.

   (b) If a divides b and b divides a, then $b = a$ or $b = -a$.

   (c) If a divides b and b divides c, then a divides c.

3. **[Falkner Section 5 Exercise 5]** Prove by induction that for each $n \in \mathbb{N}$, 5 divides $7^n - 2^n$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 5 Exercise 3(c)]** Follow the outline given in the book to find a "geometric" proof for the forumula $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$.

2. **[Falkner Section 5 Exercise 6]** Prove that for each $x \in \mathbb{Z}$, 6 divides $x^3 - x$.

   [hint: First use induction to handle the case where $x \in \mathbb{N}$.]