Homework 12

Math 3345 – Spring 2024 – Kutler

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. (a) Let n be an integer with $n > 1$. Prove that n is prime if and only if for every prime p such that $p^2 \leq n$, p does not divide n.
   (b) Use part (a) to prove that 29 is prime.
   (c) Use part (a) to prove that 101 is prime.
2. Let $a, b, q, r \in \mathbb{Z}$ such that $a = bq + r$.
   (a) Let $d \in \mathbb{N}$. Prove that d is a common divisor of a and b if and only if d is a common divisor of b and r.
   (b) Use part (a) to conclude that $\gcd(a, b) = \gcd(b, r)$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Let n be an integer. Prove that if $3|n^2$, then $3|n$.
2. Find an integer n such that $4|n^2$ but $4 \nmid n$.