

**HOMEWORK 13**  
MATH 3345 – SPRING 2024 – KUTLER

**Exercises**

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. (a) Let  $x \in \mathbb{Z}$  and let  $p$  be a prime number. Prove that

$$\gcd(p, x) = \begin{cases} p & \text{if } p|x; \\ 1 & \text{if } p \nmid x. \end{cases}$$

- (b) Let  $p$  and  $q$  be distinct prime numbers (i.e.,  $p \neq q$ ). Use part (a) to show that  $\gcd(p, q) = 1$ .

2. Use the Euclidean algorithm to compute the following.

- (a)  $\gcd(36, 22)$
- (b)  $\gcd(96, 112)$
- (c)  $\gcd(162, 31)$
- (d)  $\gcd(-15, 45)$

3. (a) Use the Euclidean algorithm to compute  $\gcd(350, 182)$ .

- (b) Find integers  $x_1$  and  $y_1$  such that  $350x_1 + 182y_1 = 14$ .

- (c) Find integers  $x_2$  and  $y_2$  such that  $350x_2 + 182y_2 = 42$ . [HINT:  $42 = 3 \cdot 14$ .]

- (d) Prove that there do not exist integers  $x$  and  $y$  such that  $350x + 182y = 16$ .

**Practice Problems**

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Let  $a \in \mathbb{Z}$ . Prove that

(a)  $\gcd(a, 1) = 1$ .

(b)  $\gcd(a, 0) = |a|$ .

2. Let  $a, b \in \mathbb{Z}$  be integers which are not both zero. Prove that  $\gcd(a, b) = \gcd(|a|, |b|)$ .

3. Let  $a, b \in \mathbb{Z}$  with  $b \neq 0$ . Prove that the fraction  $\frac{a}{b}$  is in lowest terms if and only if  $\gcd(a, b) = 1$ .