

HOMEWORK 17  
MATH 3345 – SPRING 2024 – KUTLER

**Exercises**

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 8 Exercise 9]** Let  $a, b \in \mathbb{R}$ . Suppose  $a \geq 0$  and  $b \geq 0$ . Prove that:
  - (a) If  $a < b$ , then  $a^2 < b^2$ . [Use basic properties of inequalities.]
  - (b) If  $a^2 \leq b^2$ , then  $a \leq b$ . [Do not use square roots. Use part (a).]
  - (c) If  $a^2 < b^2$ , then  $a < b$ . [Again, do not use square roots. Use part (b).]
  - (d) If  $a < b$ , then  $\sqrt{a} < \sqrt{b}$ . [Use part (c).]
2. **[Falkner Section 10 Exercise 1]** Which of the sets  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$  below are the same?

$$A = \{3\}, \quad B = \{2, 4\}, \quad C = \{x \mid x \text{ is prime, } x \text{ is odd, and } x < 5\},$$
$$D = \{x - 1 \mid x \text{ is prime, } x \text{ is odd, and } x \leq 5\}, \quad E = \{x^2 + 2 \mid x \in \{-1, 1\}\}.$$

Also, how many distinct sets are named here?

**Practice Problems**

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 8 Exercise 11]** Let  $a, b \in \mathbb{R}$ . Suppose  $a > 0$  and  $b > 0$ . Prove that  $\sqrt{a+b} < \sqrt{a} + \sqrt{b}$ . [HINT: Use Exercise 1 above.]
2. **[Falkner Section 8 Exercise 10]** Let  $a, b \in \mathbb{R}$ . Prove that if  $0 < a < b$ , then  $a < \sqrt{ab} < b$ .
3. **[Falkner Section 4 Exercise 12]** Show that for each real number  $x$ ,  $\pi + x$  is irrational or  $\pi + x$  is rational.