Homework 18

Math 3345 – Spring 2024 – Kutler

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 10 Exercise 6]**
   (a) Find a set A and a set B such that $A \notin B$ and $A \not\subseteq B$.
   (b) Find a set A and a set B such that $A \in B$ and $A \subseteq B$.
2. **[Falkner Section 10 Exercise 7]** Let $A = \{1, \{4, 7\}, 9\}$ and $B = \{\{1, 4\}, 7, 9\}$. Find $A \cup B$, $A \cap B$, $A \setminus B$, and $B \setminus A$.
3. **[Falkner Section 10 Exercise 8]** Let A and B be sets. Show that $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 10 Exercise 4]** Which of the following set notations denote the empty set?
   (a) $\{z \mid z \text{ is a horse and } z \text{ has 6 legs}\}$.
   (b) $\{a \in \mathbb{R} \mid a^2 + 2a + 2 = 0\}$.
   (c) $\{n \in \mathbb{N} \mid n^2 + n + 11 \text{ is not prime}\}$.
2. Let A and B be sets. Prove that
$$A \cup B = B \cup A \qquad \text{and} \qquad A \cap B = B \cap A.$$
3. Let A, B, and C be sets. Prove that
$$(A \cup B) \cup C = A \cup (B \cup C) \qquad \text{and} \qquad (A \cap B) \cap C = A \cap (B \cap C).$$
4. Give an example of sets A, B, and C such that
$$(A \cup B) \cap C \neq A \cup (B \cap C).$$