Homework 19

Math 3345 – Spring 2024 – Kutler

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 10 Exercise 9]** Let A, B, and C be sets. Suppose $C \subseteq A$ and $C \subseteq B$. Show that $C \subseteq A \cap B$.
2. **[Falkner Section 10 Exercise 10]** Let A and B be sets. Show that $A \subseteq B$ if and only if $A \cap B = A$.
3. **[Falkner Section 10 Exercise 11]** Let A and B be sets. Show that $A \subseteq B$ if and only if $A \setminus B = \varnothing$.
4. **[Falkner Section 10 Exercise 15 – modified]** Let S, A, and B be sets.
   (a) Prove that $S \setminus (A \setminus B) = (S \setminus A) \cup (S \cap B)$.
   (b) Use part (a) to deduce that $A \setminus (A \setminus B) = A \cap B$.
   (c) Use part (a) to deduce that $B \setminus (A \setminus B) = B$

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 10 Exercise 3]** Use set-builder notation to describe the sets
   $$A = \big\{\{1\}, \{2\}, \{3\}, \dots\big\}$$
   and
   $$B = \big\{\{1, 2, 3, \dots\}, \{2, 4, 6, \dots\}, \{3, 6, 9, \dots\}, \dots\big\}.$$
   (Of course, you will need to make reasonable assumptions about the patterns in these examples.)
2. **[Falkner Section 10 Exercise 5]** Let A be a set such that for each set B, we have $A \subseteq B$. Show that $A = \varnothing$.
3. **[Falkner Section 10 Exercise 12]** Prove Proposition 10.18(b): Let A and B be sets and let x be any object. Then
   $$x \notin A \cap B \text{ if and only if } x \notin A \text{ or } x \notin B.$$