Homework 5

Math 3345 – Spring 2024 – Kutler

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 2 Exercise 17]** Use the method of conditional proof to explain why the sentence
$$(P \Rightarrow Q) \Rightarrow \big\{[P \Rightarrow (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)\big\}$$
is a tautology. **You do NOT need to use the book's method of "discharging assumptions."**

2. **[Falkner Section 3 Exercise 1]** For each of the following sentences, write out what it means in words, state whether it is true or false, and prove your statement.

   (a) $(\exists x \in \mathbb{R})(2x + 7 = 3)$.
   (b) $(\forall x \in \mathbb{R})(2x + 7 = 3)$.
   (c) $(\exists x > 0)(2x + 7 = 3)$.
   (d) $(\forall x > 0)(2x + 7 = 3)$.
   (e) $(\exists x \in \mathbb{R})(x^2 - 4x + 3 > 0)$.
   (f) $(\forall x \in \mathbb{R})(x^2 - 4x + 3 > 0)$.
   (g) $(\exists x \geq 7)(x^2 - 4x + 3 > 0)$.
   (h) $(\forall x \geq 7)(x^2 - 4x + 3 > 0)$.
   (i) $(\forall x \in \mathbb{R})(x^2 - 2x + 2 > 0)$.
   (j) $(\forall x \geq 0)(\sqrt{x+3} = \sqrt{x} + \sqrt{3})$.
   (k) $(\exists x \geq 0)(\sqrt{x+3} = \sqrt{x} + \sqrt{3})$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 2 Exercise 22]** Let A be the sentence
$(P \Rightarrow Q) \Rightarrow \big\{[P \Rightarrow (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)\big\}$. We saw in Exercise 17 that A is a tautology. Let B be the converse of A. Write out B in terms of P, Q, and R. Then show that B is not a tautology, by finding a combination of truth values for P, Q, and R that makes B false. You should be able to do this without writing out a truth table.