Homework 6

Math 3345 – Spring 2024 – Kutler

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 3 Exercise 7]** Let P be the sentence

$$(\exists x \in \mathbb{R})\big(x \geq 0 \text{ and } \sqrt{x+2} < \sqrt{x} + \sqrt{2}\big).$$

   (a) Use one of the generalized De Morgan's laws and one of the ordinary De Morgan's laws to show that $\neg P$ is logically equivalent to

$$(\forall x \in \mathbb{R})\big(x < 0 \text{ or } \sqrt{x+2} \geq \sqrt{x} + \sqrt{2}\big).$$

   (b) Is P true or false? Provide a proof for your answer.

2. **[Falkner Section 3 Exercise 10]** For each of the following sentences, write out what it means in words, state whether it is true or false, and prove your statement.

   (a) $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(x + y = x)$.

   (b) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x + y = x)$.

   (c) $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(x + y = 0)$.

   (d) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x + y = 0)$.

   (e) $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(xy = 1)$.

   (f) $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(xy = 1)$.

3. **[Falkner Section 5 Exercise 2]** Prove by induction that for each $n \in \mathbb{N}$,

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 3 Exercise 8]** Which of the variables x and y is free in the sentence $P(x, y)$. Answer the same question about each of the four sentences $(\exists y)P(x, y)$, $(\forall x)(\exists y)P(x, y)$, $(\forall x)P(x, y)$, and $(\exists y)(\forall x)P(x, y)$.

2. **[Falkner Section 3 Exercise 9]** See book for problem statement.