

HOMEWORK 7
MATH 3345 – SPRING 2024 – KUTLER

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. Let $A(x, y)$ be the conditional sentence

$$\text{If } x = 3 \text{ and } y = 5, \text{ then } xy = 15.$$

Although most people probably read this sentence as a straightforwardly true statement, it is technically a function of the variables x and y (it's just that " $x = 3$ and $y = 5$ " is false for almost all values of x and y). To produce an unambiguously true or false sentence, we should bound these variables with quantifiers.

- (a) Is $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R}) A(x, y)$ true or false? Explain your answer.
- (b) Use one of the generalized De Morgan's laws to write the negation of the sentence $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R}) A(x, y)$.
- (c) Let $B(x, y)$ be the converse of $A(x, y)$. Is $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R}) B(x, y)$ true or false? Explain your answer.
2. **[Falkner Section 3 Exercise 14]** For each of the following sentences, write out what it means in words, state whether it is true or false, and prove your answer.
- (a) $(\exists! x \in \mathbb{R})(2x + 7 = 3)$.
- (b) $(\exists! x \in \mathbb{R})(x^2 - 4x + 3 < 0)$.
- (c) $(\exists! x \in \mathbb{Z})(x^2 - 4x + 3 < 0)$.
- (d) $(\exists! x \in \mathbb{R})(x^2 - 4x + 4 = 0)$.
- (e) $(\exists! x \in \mathbb{R})(x^2 - 4x + 5 = 0)$.
- (f) $(\forall x \in \mathbb{R})(\exists! y \in \mathbb{R})(x + y = 0)$.
- (g) $(\forall x \in \mathbb{R})(\exists! y \in \mathbb{R})(xy = 1)$.
- (h) $(\forall x \in \mathbb{R})[\text{if } x \neq 0, \text{ then } (\exists! y \in \mathbb{R})(xy = 1)]$.
- (i) $(\forall x \in \mathbb{R})(\exists! y \in \mathbb{R})(xy = 0)$.
- (j) $(\forall x \in \mathbb{R})[\text{if } x \neq 0, \text{ then } (\exists! y \in \mathbb{R})(xy = 0)]$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Read the (incorrect) proof given in [**Falkner Section 5 Exercise 8**], which purports to prove by induction that all horses have the same color.
 - (a) Modify this proof to obtain (incorrect) proofs of the following (false) statements:
 - i. All students at OSU have the same favorite food.
 - ii. All songs are in the same key.
 - iii. All rivers flow in the same direction.
 - iv. All mountains have the same height.
 - (b) Each of these proofs makes the same error. What is it?