Homework 8
Math 3345 – Spring 2024 – Kutler

Exercises

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. In class, we saw a **correct** proof of the following statement: For each $n \in \mathbb{N}$,
$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$
Consider the **incorrect** proof of the same statement given below.

> *Proof.* **Base Case:** Since $1 = \frac{1 \cdot 2}{2}$, $P(1)$ is true.
>
> **Inductive Step:** Let $n \in \mathbb{N}$. Then
> $$\begin{aligned} 1 + 2 + \cdots + n + (n+1) &= \frac{(n+1)(n+2)}{2} \\ \frac{n(n+1)}{2} + (n+1) &= \frac{(n+1)(n+2)}{2} \\ n+1 &= \frac{(n+1)(n+2)}{2} - \frac{n(n+1)}{2} \\ n+1 &= \left(\frac{n+1}{2}\right)\big((n+2) - n\big) \\ n+1 &= n+1. \end{aligned}$$
>
> Thus, $P(n+1)$ is true. $\square$

Explain, in complete sentences, what is wrong with this proof. How would you fix it?

2. [**Falkner Section 4 Exercise 1**] Let x, y, and z be integers.
   (a) Prove that if x is even and y is even, then $x + y$ is even.
   (b) Prove that if x is even and y is odd, then $x + y$ is odd.
   (c) Suppose x, y, and z are odd. Is $x+y+z$ odd? Or is $x+y+z$ even? Explain your answer. You should not have to use the definitions of odd and even. Instead, you should be able to answer this part by combining one of parts (a) and (b) with Example 4.5.

3. **[Falkner Section 4 Exercise 2]** Let x, y, and z be integers.
   (a) Prove that if x is odd and y is odd, then xy is odd.
   (b) Suppose x, y, and z are odd. Is xyz odd? Or is xyz even? You should not have to use the definitions of odd and even. Instead, you should be able to answer this part by applying part (a).

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 4 Exercise 3]** Let x be an integer. Prove that $x(x+1)$ is even.
2. Let $r \neq 1$ be a real number. Prove that for every $n \in \mathbb{N}$,

$$1 + r + r^2 + \cdots + r^{n-1} = \frac{r^n - 1}{r - 1}.$$

   This is the (finite) **geometric series formula**.