

HOMEWORK 9  
MATH 3345 – SPRING 2024 – KUTLER

**Exercises**

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 4 Exercise 5]** Let  $x$  and  $y$  be integers. Prove the following statements.
  - (a) If  $xy$  is even, then  $x$  is even or  $y$  is even.
  - (b) If  $xy$  is odd, then  $x$  is odd and  $y$  is odd.
2. **[Falkner Section 4 Exercise 6]** Let  $a$  be an integer. Use the results of the previous exercise to prove the following statements.
  - (a) If  $a^2$  is even, then  $a$  is even.
  - (b) If  $a^2$  is odd, then  $a$  is odd.

3. **[Integers Exercise 1]** Prove Lemma 4: For any  $a \in \mathbb{Z}$ ,  $-(-a) = a$ .

You may **only** assume the axioms for the integers and Lemmas 1–3 on the Integers handout. You may find the Additive Inverses axiom and Lemma 2 (Uniqueness of Additive Inverses) particularly useful.

4. **[Integers Exercise 2]** Prove Lemma 7: For any  $a, b \in \mathbb{Z}$ , if  $a \cdot b = 0$ , then  $a = 0$  or  $b = 0$ .

You may use only the axioms for the integers and Lemmas 1 through 6. [HINT: Prove the contrapositive statement. If an integer  $x$  is not zero, then by the Trichotomy axiom, either  $x \in \mathbb{N}$  or  $-x \in \mathbb{N}$ . Now, consider cases and use the Positive Closure axiom.]

**Practice Problems**

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Integers Exercise 3]** Prove Lemma 9: For any  $a, b \in \mathbb{Z}$ , exactly one of the following is true: (i)  $a < b$ , or (ii)  $a = b$ , or (iii)  $b < a$ .

[HINT: You will need to use the Trichotomy axiom, the Distributive Law, and Lemmas 4 and 5.]