

HOMEWORK 10  
MATH 3345 – AUTUMN 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Integers Exercise 1]** Prove Lemma 4: For any  $a \in \mathbb{Z}$ ,  $-(-a) = a$ .

You may **only** assume the axioms for the integers and Lemmas 1–3 on the Integers handout. You may find the Additive Inverses axiom and Lemma 2 (Uniqueness of Additive Inverses) particularly useful.

2. **[Integers Exercise 2]** Prove Lemma 7: For any  $a, b \in \mathbb{Z}$ , if  $a \cdot b = 0$ , then  $a = 0$  or  $b = 0$ .

You may use only the axioms for the integers and Lemmas 1 through 6. [HINT: Prove the contrapositive statement. If an integer  $x$  is not zero, then by the Trichotomy axiom, either  $x \in \mathbb{N}$  or  $-x \in \mathbb{N}$ . Now, consider cases and use the Positive Closure axiom.]

3. **[MOVED TO HW 11]**

**Practice Problems**

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Integers Exercise 3]** Prove Lemma 9: For any  $a, b \in \mathbb{Z}$ , exactly one of the following is true: (i)  $a < b$ , or (ii)  $a = b$ , or (iii)  $b < a$ .

[HINT: You will need to use the Trichotomy axiom, the Distributive Law, and Lemmas 4 and 5.]

2. **[Falkner Section 4 Exercise 7]** Explain what is wrong with the following “proof” that  $-3 = 5$ : Suppose that  $-3 = 5$ . Then  $-3 - 1 = 5 - 1$ . Hence,  $-4 = 4$ . But then  $(-4)^2 = 4^2$ . In other words,  $16 = 16$ . This is true. Hence, our assumption that  $-3 = 5$  is correct. ■