

**HOMEWORK 11**  
MATH 3345 – AUTUMN 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 4 Exercise 14]** Let  $a, b, c \in \mathbb{Z}$ . Prove the following statements.
  - (a)  $a$  divides  $a$ .
  - (b) If  $a$  divides  $b$  and  $b$  divides  $a$ , then  $b = a$  or  $b = -a$ .
  - (c) If  $a$  divides  $b$  and  $b$  divides  $c$ , then  $a$  divides  $c$ .
2. **[Falkner Section 5 Exercise 5]** Prove by induction that for each  $n \in \mathbb{N}$ , 5 divides  $7^n - 2^n$ .
3. **[The sieve of Eratosthenes]**
  - (a) Write all of the integers from 1 to 100 (perhaps as a  $10 \times 10$  array), and do the following.
    - i. Cross out the number 1.
    - ii. Circle the number 2, then cross out all other numbers divisible by 2.
    - iii. Circle the number 3, then cross out all other numbers divisible by 3.
    - iv. Repeat this procedure: Circle the smallest number  $d$  which is not crossed out, then cross out all other numbers divisible by  $d$ . Stop once every integer from 1 to 100 is either circled or crossed out.
  - (b) Explain why the circled numbers are all of the primes less than 100.

**Practice Problems**

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Explain why 0 is the only integer with infinitely many divisors.
2. **[Falkner Section 4 Exercise 13]** Let  $a, b, c \in \mathbb{Z}$ . Prove the following statements.
  - (a) If  $a|b$  and  $a|c$ , then  $a|(b + c)$  and  $a|(b - c)$ .
  - (b) If  $a|b$  or  $a|c$ , then  $a|bc$ .
  - (c) If  $a|b$ , then  $a|(-b)$ .
  - (d) If  $a|b$ , then  $(-a)|b$ .