

HOMEWORK 13
MATH 3345 – AUTUMN 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 8 Exercise 9]** Let $a, b \in \mathbb{R}$. Suppose $a \geq 0$ and $b \geq 0$. Prove that:
 - (a) If $a < b$, then $a^2 < b^2$. [Use basic properties of inequalities.]
 - (b) If $a^2 \leq b^2$, then $a \leq b$. [Do not use square roots. Use part (a).]
 - (c) If $a^2 < b^2$, then $a < b$. [Again, do not use square roots. Use part (b).]
 - (d) If $a < b$, then $\sqrt{a} < \sqrt{b}$. [Use part (c).]
2. **[Falkner Section 8 Exercise 11]** Let $a, b \in \mathbb{R}$. Suppose $a > 0$ and $b > 0$. Prove that $\sqrt{a+b} < \sqrt{a} + \sqrt{b}$. [HINT: Use the previous exercise.]
3. **[Falkner Section 4 Exercise 8]** Let x and y be rational numbers. Prove the following statements.
 - (a) $-y$ is a rational number.
 - (b) $x - y$ is a rational number.
 - (c) xy is a rational number.
 - (d) If $y \neq 0$, then $1/y$ is a rational number.
 - (e) If $y \neq 0$, then x/y is a rational number.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 4 Exercise 12]** Show that for each real number x , $\pi + x$ is irrational or $\pi + x$ is rational.
2. **[Falkner Section 8 Exercise 10]** Let $a, b \in \mathbb{R}$. Prove that if $0 < a < b$, then $a < \sqrt{ab} < b$.