Homework 20

Math 3345 – Autumn 2022 – Kutler

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 10 Exercise 11]** Let A and B be sets. Show that $A \subseteq B$ if and only if $A \setminus B = \varnothing$.

2. **[Falkner Section 10 Exercise 15 – modified]** Let S, A, and B be sets.

   (a) Prove that $S \setminus (A \setminus B) = (S \setminus A) \cup (S \cap B)$.

   (b) Use part (a) to deduce that $A \setminus (A \setminus B) = A \cap B$.

   (c) Use part (a) to deduce that $B \setminus (A \setminus B) = B$

3. **[Falkner Section 10 Exercise 19 – modified]** Let A, B, and X be sets.

   (a) Prove that if $A \subseteq B$, then $X \setminus B \subseteq X \setminus A$.

   (b) Prove that $A \subseteq X$ if and only if $A = X \setminus (X \setminus A)$. [HINT: Use the previous problem to express $X \setminus (X \setminus A)$ in a simpler form.]

   (c) Suppose $A \subseteq X$. Prove that if $X \setminus B \subseteq X \setminus A$, then $A \subseteq B$.

   (d) Show, by giving an example, that the implication

   $$\text{if } X \setminus B \subseteq X \setminus A, \text{ then } A \subseteq B$$

   may be **false** if $A \not\subseteq X$.
   That is, give an example of sets A, B, and X such that $X \setminus B \subseteq X \setminus A$ and $A \not\subseteq B$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 10 Exercise 3]** Use set-builder notation to describe the sets

   $$A = \big\{\{1\}, \{2\}, \{3\}, \dots\big\}$$

   and

   $$B = \big\{\{1, 2, 3, \dots\}, \{2, 4, 6, \dots\}, \{3, 6, 9, \dots\}, \dots\big\}.$$

   (Of course, you will need to make reasonable assumptions about the patterns in these examples.)

2. [**Falkner Section 10 Exercise 5**] Let A be a set such that for each set B, we have $A \subseteq B$. Show that $A = \varnothing$.

3. [**Falkner Section 10 Exercise 12**] Prove Proposition 10.18(b): Let A and B be sets and let x be any object. Then

$$x \notin A \cap B \text{ if and only if } x \notin A \text{ or } x \notin B.$$

4. [**Falkner Section 10 Exercise 24**] Prove Proposition 10.34(b): Let $\mathscr{A}$ be a nonempty set of sets and let X be any object. Then

$$x \notin \left(\bigcap_{A \in \mathscr{A}} A \right) \text{ if and only if there exists } A \in \mathscr{A} \text{ such that } x \notin A.$$