Homework 21

Math 3345 – Autumn 2022 – Kutler

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 10 Exercise 26]** Prove Theorem 10.36(b): Let S be a set and let $\mathscr{A}$ be a nonempty set of sets. Then
$$S \cup \left(\bigcap_{A \in \mathscr{A}} A \right) = \bigcap_{A \in \mathscr{A}} (S \cup A).$$

2. **[Falkner Section 10 Exercise 33(a)–(d)]** Let A, B, C, and D be sets.
   (a) Prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.
   (b) Prove that $(A \cup B) \times C = (A \times C) \cup (B \times C)$ and $A \times (B \cup C) = (A \times B) \cup (A \times C)$.
   (c) Prove that $(C \times D) \setminus (A \times B) = E \cup F$, where $E = (C \setminus A) \times D$ and $F = C \times (D \setminus B)$.
   (d) In the special case where $A = [1, 3] = B$ and $C = [2, 4] = D$, draw a picture to illustrate the result you proved for the general case in part (c).

3. **[Falkner Section 10 Exercise 34 – modified]**
   (a) Let A be a set. Prove that $A \times \varnothing = \varnothing$.
   (b) Let A and B be sets. Deduce that $A \times \varnothing = B \times \varnothing$.
   (c) Let A, B, and C be sets, and suppose that $C \neq \varnothing$. Prove that if $A \times C = B \times C$, then $A = B$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 10 Exercise 27]** Let A be a set and let $\mathscr{B}$ be a nonempty set of sets. Show that:
   (a) $A \cup \left(\bigcup_{B \in \mathscr{B}} B \right) = \bigcup_{B \in \mathscr{A}} (A \cup B)$
   (b) $A \cap \left(\bigcap_{B \in \mathscr{B}} B \right) = \bigcap_{B \in \mathscr{A}} (A \cap B)$

2. **[Falkner Section 10 Exercise 32]** Sketch the rectangle $[1, 4] \times [2, 3]$ in the coordinate plane. (Shade the set of points that belong to this rectangle.