Homework 5

Math 3345 – Autumn 2022 – Kutler

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 2 Exercise 9]** Let $P \operatorname{xor} Q$ mean "P exclusive or Q." In other words, $P \operatorname{xor} Q$ should be true just when **exactly one** of P or Q is true.
   (a) Write out the truth table for $P \operatorname{xor} Q$.
   (b) Show by a truth table that $P \operatorname{xor} Q$ is logically equivalent to $(P \wedge \neg Q) \vee (Q \wedge \neg P)$.
   (c) Show by truth tables that the following four sentences are logically equivalent:
   $$P \operatorname{xor} Q, \qquad \neg(P \Leftrightarrow Q), \qquad (\neg P) \Leftrightarrow Q, \qquad P \Leftrightarrow (\neg Q).$$
   (d) Show by a truth table that $(\neg P) \Leftrightarrow (\neg Q)$ is logically equivalent to $P \Leftrightarrow Q$.

2. **[Falkner Section 3 Exercise 1]** For each of the following sentences, write out what it means in words, state whether it is true or false, and prove your statement.
   (a) $(\exists x \in \mathbb{R})(2x + 7 = 3)$.
   (b) $(\forall x \in \mathbb{R})(2x + 7 = 3)$.
   (c) $(\exists x > 0)(2x + 7 = 3)$.
   (d) $(\forall x > 0)(2x + 7 = 3)$.
   (e) $(\exists x \in \mathbb{R})(x^2 - 4x + 3 > 0)$.
   (f) $(\forall x \in \mathbb{R})(x^2 - 4x + 3 > 0)$.
   (g) $(\exists x \geq 7)(x^2 - 4x + 3 > 0)$.
   (h) $(\forall x \geq 7)(x^2 - 4x + 3 > 0)$.
   (i) $(\forall x \in \mathbb{R})(x^2 - 2x + 2 > 0)$.
   (j) $(\forall x \geq 0)(\sqrt{x+3} = \sqrt{x} + \sqrt{3})$.
   (k) $(\exists x \geq 0)(\sqrt{x+3} = \sqrt{x} + \sqrt{3})$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. (a) [**Falkner Section 2 Exercise 4**] Suppose that $P \vee Q$ is true and $\neg Q$ is true. Explain why it follows that P must be true.

   (b) Prove that the conditional sentence

   $$\big[(P \vee Q) \wedge \neg Q\big] \;\Rightarrow\; P$$

   is a tautology (that is, it is true for all possible truth values of P and Q).

   Do not use a truth table. Rather, use your work from part (a) to write a conditional proof.