

HOMEWORK 6  
MATH 3345 – AUTUMN 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 2 Exercise 17]** Use the method of conditional proof to explain why the sentence

$$(P \Rightarrow Q) \Rightarrow \{[P \Rightarrow (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)\}$$

is a tautology. **You do NOT need to use the book’s method of “discharging assumptions.” However, please be clear about which assumptions you are making, and why you may make these assumptions.**

2. Let  $A(x, y)$  be the conditional sentence

$$\text{If } x = 3 \text{ and } y = 5, \text{ then } xy = 15.$$

- (a) Use one of the generalized De Morgan’s laws to write the negation of the sentence  $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R}) A(x, y)$ .
- (b) Is  $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R}) A(x, y)$  true or false? Explain your answer.
- (c) Let  $B(x, y)$  be the converse of  $A(x, y)$ . Is  $(\forall x \in \mathbb{R})(\forall y \in \mathbb{R}) B(x, y)$  true or false? Explain your answer.
3. **[Falkner Section 3 Exercise 10]** For each of the following sentences, write out what it means in words, state whether it is true or false, and prove your statement.
- (a)  $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(x + y = x)$ .
- (b)  $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x + y = x)$ .
- (c)  $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(x + y = 0)$ .
- (d)  $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x + y = 0)$ .
- (e)  $(\exists y \in \mathbb{R})(\forall x \in \mathbb{R})(xy = 1)$ .
- (f)  $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(xy = 1)$ .

## Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 2 Exercise 22]** Let  $A$  be the sentence  $(P \Rightarrow Q) \Rightarrow \{[P \Rightarrow (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)\}$ . We saw in Exercise 17 that  $A$  is a tautology. Let  $B$  be the converse of  $A$ . Write out  $B$  in terms of  $P$ ,  $Q$ , and  $R$ . Then show that  $B$  is not a tautology, by finding a combination of truth values for  $P$ ,  $Q$ , and  $R$  that makes  $B$  false. You should be able to do this without writing out a truth table.
2. **[Falkner Section 3 Exercise 9]** See book for problem statement.