Homework 9

Math 3345 – Autumn 2022 – Kutler

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 4 Exercise 5]** Let x and y be integers. Prove the following statements.
   (a) If xy is even, then x is even or y is even.
   (b) If xy is odd, then x is odd and y is odd.
2. **[Falkner Section 4 Exercise 6]** Let a be an integer. Use the results of the previous exercise to prove the following statements.
   (a) If a^2 is even, then a is even.
   (b) If a^2 is odd, then a is odd.
3. **[Falkner Section 5 Exercise 3(a)(b)]**
   (a) Prove by induction that for each $n \in \mathbb{N}$,
   $$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$$
   (b) Explain why it follows from part (a) and Exercise 1 (cf. notes from Lecture 10) that for each $n \in \mathbb{N}$,
   $$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2.$$

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 5 Exercise 3(c)]** Follow the outline given in the book to find a "geometric" proof for the forumula $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$.
2. **[Falkner Section 5 Exercise 6]** Prove that for each $x \in \mathbb{Z}$, 6 divides $x^3 - x$.
   [HINT: First use induction to handle the case where $x \in \mathbb{N}$.]