Let P, Q, R, ... stand for sentences.

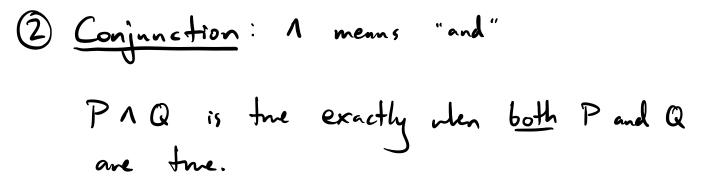
$$Ex: P = "It is Friday."$$

 $Q = "We're having fun in Math 3345."$
 $\neg P, PAQ, P \Rightarrow Q, etc.$

Ex: What is ¬(¬P)? Make another truth tuble:

$$\begin{array}{c|c} P & \neg P & \neg (\neg P) \\ \hline T & F & T \\ F & T & F \end{array}$$

If P is the, then
$$\neg P$$
 is folse. If $\neg P$ is
folse, then $\neg (\neg P)$ is thre. So is P is the
then $\neg (\neg P)$ is the.
Similarly, if P is folse, then $\neg (\neg P)$ is folse.
So P and $\neg (\neg P)$ always have the
same finth value. We say they are
logically equivalent and write
$$P \equiv \neg (\neg P).$$



Ρ	Q	PAQ
Т	T	Т
Т	F	F
F	Т	F
F	F	F

<u>P</u>	Q	PVQ	
Т	T	Т	
Т	F	Т	
F	Т	Т	
F	F	F	

$$\underbrace{N_{o}te}_{P \vee Q} = Q \vee P$$
$$P \vee (Q \vee R) = (P \vee Q) \vee R$$

How do the operations
$$\neg$$
, \land , \lor interact with one another?

Then PAQ is "mis even and n is odd."

This becomes false if m is not even OR n is not odd.

= - P V - Q

Thm (De Morgan's Laws) Let P and Q be sentences. Then

Proof of (a):									
By truth table:									
P	Q	PAQ	- (P1Q)	٦P	¬ Q	-PV-Q			
Т	Т	Т	F	11	Ţ	Ч			
Т	F	F	Т	-	Т	Т			
F	Т	Ч	Т	Т	F	Т			
F	F	F	Т	T	Т	Т			

So we see ~ (PAQ) = ~ PV ~Q.

In words:

We wish to show $\neg(PAQ)$ always has the same truth value as $\neg PV \neg Q$.

First, suppose $\neg(PAQ)$ is true. Then PAQis false, so at least one of P or Q is false.

But this means at least one of $\neg P$ or $\neg Q$ is true, so $\neg P \lor \neg Q$ is true.

Next, suppose -(PAQ) is fulse. Then PAQ is true, so both P and Q are true. Now, both -P and -Q will be fulse, meaning -PV-Q is fulse as well.

(b) HW 1