More tantologies

Ex: PV-P is a tantology. "The law of the excluded middle."

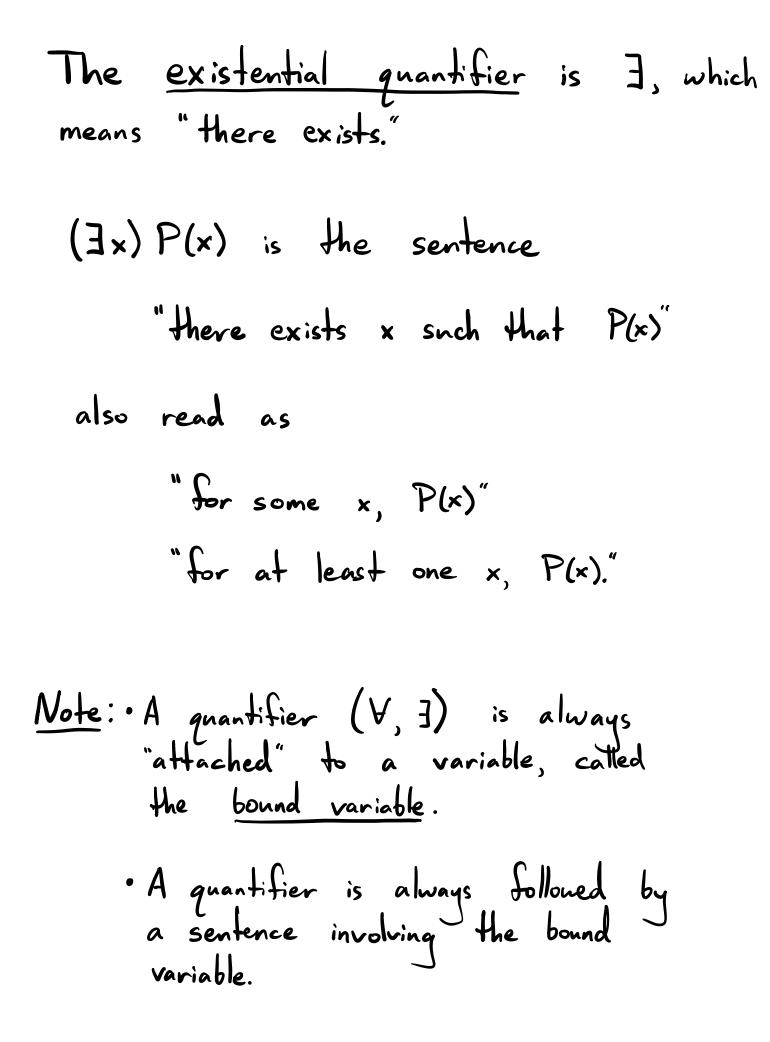
Ex: Show that the sentence $\left[(P \Rightarrow Q) \land (Q \Rightarrow R) \right] \Rightarrow (P \Rightarrow R)$ is a tantology. "Hypothetical syllogism" Proof: Suppose $(P \Rightarrow Q) \land (Q \Rightarrow R)$ is true. Thus, $P \Rightarrow Q$ and $Q \Rightarrow R$ are both true. We want to argue that $P \Rightarrow R$ must be true.

Assume P is true. If not,
$$P \Rightarrow Q$$
 is true vacually
Since $P \Rightarrow Q$ and P are both
true, Q must be true.
Since $Q \Rightarrow R$ and Q are both
true, R must be true.
Therefore, $P \Rightarrow R$ is true, as
desired.

Recall: The truth value of the sentence
"If
$$xy > 0$$
, then $x > 0$ and $y > 0$ "
depends on the numerical values of x
and y.
We said it is false because it can be
false (e.g. $x=-1$, $y=-1$), but it can also
be true (e.g. $x=1$, $y=1$ or $x=-1$, $y=1$).
Quantifiers let us discuss such situations.

The <u>universal quantifier</u> is V, which means "for all."

If
$$P(x)$$
 is a sentence involving the
variable x, then $(\forall x) P(x)$ is the
sentence
"for all x, $P(x)$ "
also read as
"for a p(x)"



Ex: Let
$$P(x)$$
 be the sentence
" $(x \ge 1) \Rightarrow (x^2 \ge 1)$ "
and let $Q(x)$ be the converse
" $(x^2 \ge 1) \Rightarrow (x \ge 1)$."
Then $(\forall x) P(x)$ is true.
 $(\exists x) P(x)$ is true.
 $(\exists x) Q(x)$ is fulse.
 $(\exists x) Q(x)$ is true.
Note: We should be more careful to
specify which values the bound
variable is allowed to take on.
The above statements are correct
when x can be any real number.
To indicate this, we will write
 $(\forall x \in \mathbb{R})$ and $(\exists x \in \mathbb{R})$.