

Logical Sentences

A logical sentence is a sentence with a well-defined truth value (True or False).

Ex: "OSU is in Columbus, OH." (T)

" $2+2=5$." (F)

" $x > 7$ " (Depends on variable x)

Non-Ex: "Go outside."
"Are you cold?"

It is clear from these examples that logical sentences require precise definitions.

Propositional Calculus ← Nothing to do with differential/integral calculus

How to "build" new sentences from existing ones?

Use logical connectives.

Logical Connective	Symbol	Plain English
negation	\neg	"not"
conjunction	\wedge	"and"
disjunction	\vee	"or" (inclusive)
implication	\Rightarrow	"if-then"
biconditional	\Leftrightarrow	"if and only if"

Let P, Q, R, \dots stand for sentences.

Ex: $P =$ "It is Friday."

$Q =$ "We're having fun in Math 3345."

$\neg P, P \wedge Q, P \Rightarrow Q, \text{ etc.}$

① Negation: \neg means "not"

The negation $\neg P$ has the opposite truth value as P .

So if P is true, then $\neg P$ is false,
if P is false, then $\neg P$ is true.

Summarize this in a truth table:

P	$\neg P$
T	F
F	T

Ex: What is $\neg(\neg P)$? Make another truth table:

P	$\neg P$	$\neg(\neg P)$
T	F	T
F	T	F

So P and $\neg(\neg P)$ always have the same truth value. We say they are logically equivalent and write

$$P \equiv \neg(\neg P).$$

\uparrow
"is logically equivalent to"

② Conjunction: \wedge means "and"

$P \wedge Q$ is true exactly when both P and Q are true.

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Ex: 2 is even and 3 is odd. T
2 is even and 3 is even. F
2 is odd and 3 is odd. F