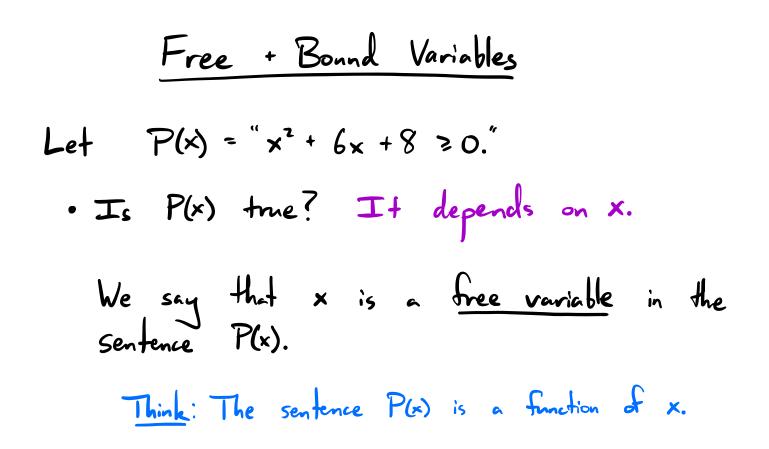
$$(d) \quad (x > 2) \iff (x^2 > 4)$$

(x) $(x>2) \Rightarrow (x^2>4)$



$$\frac{\text{Thm}}{(a)} - \left[(\forall x \in A) P(x) \right] = (\exists x \in A) (\neg P(x))$$

$$(b) - \left[(\exists x \in A) P(x) \right] = (\forall x \in A) (\neg P(x))$$

$$\frac{P_{roo}f:(n)}{Then} \quad \neg \left[(\forall x \in A) P(x) \right] \quad is \quad tme.$$

$$Then \quad (\forall x \in A) P(x) \quad is \quad fulse.$$

So there is some
$$x_0 \in A$$
 such that $P(x_0)$
is fulse, i.e. $\neg P(x_0)$ is true.
Hence $(\exists x \in A) (\neg P(x))$ is true.
Conversely, suppose $(\exists x \in A) (\neg P(x))$ is true.
Then there is $x_0 \in A$ such that $\neg P(x_0)$
is true, i.e. $P(x_0)$ is fulse.
So $(\forall x \in A) P(x)$ is fulse. Therefore,
 $\neg (\forall x \in A) P(x)$ is true.

(b) is similar (see book).

$$\frac{Th m}{Let P be a sentence not involving x.}$$
Let Q(x) be a sentence involving x.
Then
a) PA [(∃ x ∈ A) Q(x)] = (∃ x ∈ A) [PAQ(x)]
b) PV [(∀ x ∈ A) Q(x)] = (∀ x ∈ A) [PVQ(x)].

$$\frac{Proof}{2}: Omitted (see book).$$

Ex: A = B = IR, P(x,y) = "x+y=1"
i is "for every x \in IR, there exists y \in IR such that x+y=1."
True!
Proof: Let x \in IR. Set y=1-x \in IR.
Then x+y=x+(1-x)=1.

is "there exists some y i R such that for every x i R, we have x+y=1."
 False!
 Proof: Suppose y i R. Then x+y=1 is not true for every x i R, since we could take x=-y and get x+y=(-y)+y=0 \$\$1.

By Generalized DeMorgan, the negation ¬ (∃y ∈ IR)(∀x ∈ IR)(x+y=1) = (∀y ∈ IR)¬(∀x ∈ IR)(x+y=1) = (∀y ∈ R)(∃x ∈ IR)(x+y≠1) is True, and this is exactly what we proved.

To summarize:

$$\begin{array}{l}
\textcircled{O}(\forall x \in A)(\exists y \in B) P(x,y) \\
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\fbox{O}(\exists y \in B)(\forall x \in A) P(x,y) \\
\r{O}(x,y) \\
\r{O$$

In other words, (Vx & A)(Zy & B)P(x,y) is true, as desired.