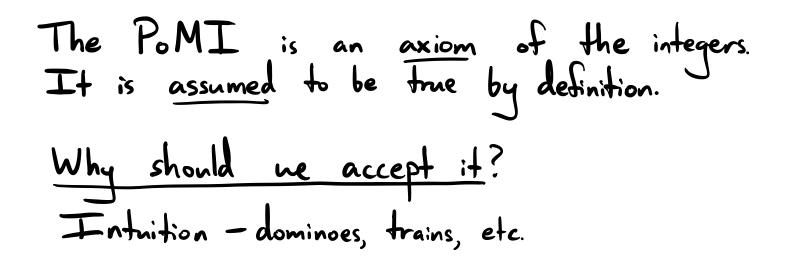
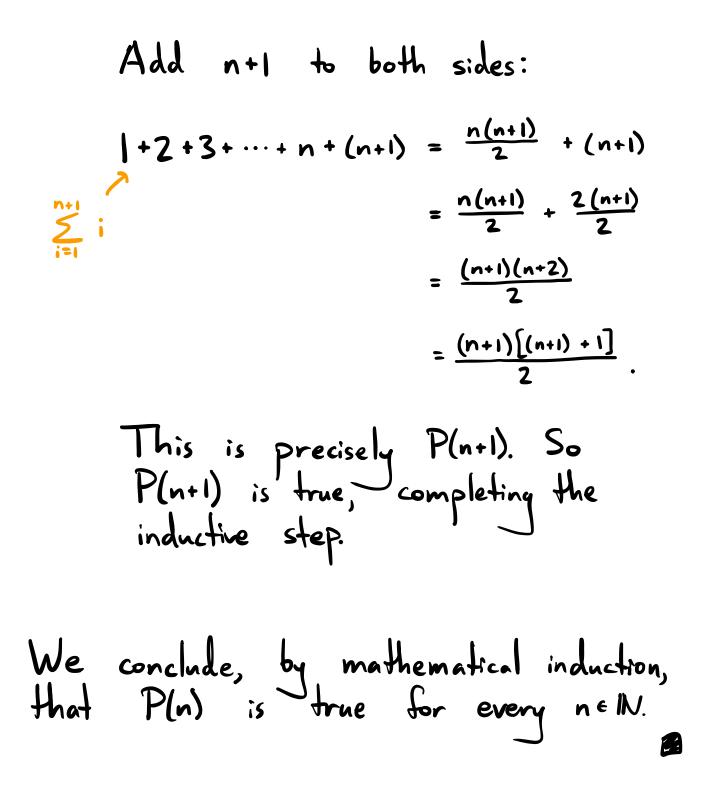
Warm-Up: Let 
$$P(x)$$
 be a sentence  
depending on  $x \in A$ . Prove that  
 $(\forall x \in A) P(x) \implies (\exists x \in A) P(x)$   
is a trantology.

The Principle of Mathematical Induction  
Let 
$$P(n)$$
 be a sentence involving  $n \in \mathbb{N}$ .  
Ex:  $P(n) = \frac{2}{12}i = \frac{n(n+1)}{2}$ 

In symbols:  $\left\{ P(1) \land \left[ (\forall n \in IN) (P(n) \Rightarrow P(n+1)) \right] \right\} \Rightarrow (\forall n \in IN) P(n)$ 



Thm: For all  $n \in \mathbb{N}$ ,  $\sum_{i=1}^{n} i = \frac{n(n+i)}{2}$ Proof: Let P(n) be  $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ . We will prove (VnEN)P(n) by induction on n. Base Case: When n=1,  $\sum_{i=1}^{n} i = 1$  and  $\frac{1(1+1)}{2} = 1$ , so  $P(1) = "\sum_{i=1}^{l} i = \frac{I(1+i)}{2}$ " is true. Inductive Step: Let n∈IN. We will prove P(n) ⇒ P(n+1) is true. Suppose P(n) is true. That is,  $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ for this particular number n.  $\int_{0}^{\infty} 1+2+3+\cdots+n = \frac{n(n+1)}{2}$ .



Then: For every 
$$n \in IN$$
,  
 $1+3+5+\cdots+(2n-1) = n^2$ .  
 $= \sum_{i=1}^{n} (2i-1)$   
Proof: We proceed by induction on  $n$ .  
Let  $P(n)$  be " $1+3+5+\cdots+(2n-1) = n^2$ ."  
Base Case: When  $n=1$ ,  $P(1)$  is  
" $1 = 1^2$ "  
which is true.  
Inductive Step: Let  $n \in IN$ . We  
wish to prove  $P(n) \Rightarrow P(n+1)$ ,

So we may assume P(n). Thus,  $1+3+5+\cdots+(2n-1)=n^2$ 

is true (for this n).

Now,  

$$1+3+5+\cdots+(2n-1)+[2(n+1)-1]$$
  
 $= n^{2}+[2n+2-1]$   
 $= n^{2}+2n+1$   
 $=(n+1)^{2}$ .  
Thus, we have shown that  
 $P(n+1)$  is true, completing the  
inductive step.  
By induction, we conclude that  
 $P(n)$  is true for all  $n \in M$ .  
Does the base case have to be  $n=1$ ?  
 $No!$ 

Ex (Exam I Review 8(e)): For every 
$$n \in IN$$
  
such that  $n > 3$ ,  $2^n < n!$   
  
$$\frac{Check}{2^3} = 8 > 6 = 3! \times 2^n < 16 < 24 = 4! < 2^5 = 32 < 120 = 5! < 2^n < 12^n < 1$$

Since 
$$2 \le 3 \le n \le n+1$$
, we have  
 $2^{n+1} \le 2n! \le (n+1)n! = (n+1)!$   
Thus,  $P(n+1)$  is true, completing  
the inductive step.  
We conclude that  $P(n)$  is true  
for every  $n \le N$  such that  $n \ge 3$ .  
Note: We could have equivalently set  
 $Q(n) = P(n+3) = "2^{n+3} \le (n+3)!"$   
and proved  $(\forall n \le N) Q(n)$  by  
induction starting at  $n=1$ .