HOMEWORK 10
MATH 3345 – SPRING 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. [**Falkner Section 5 Exercise 3(a)(b)**]

   (a) Prove by induction that for each $n \in \mathbb{N}$,

   $$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}.$$

   (b) Explain why it follows from part (a) and Exercise 1 that for each $n \in \mathbb{N}$,

   $$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2.$$

2. [**Falkner Section 4 Exercise 14**] Let $a, b, c \in \mathbb{Z}$. Prove the following statements.

   (a) a divides a.

   (b) If a divides b and b divides a, then $b = a$ or $b = -a$.

   (c) If a divides b and b divides c, then a divides c.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. [**Falkner Section 5 Exercise 3(c)**] Follow the outline in the book to find a "geometric" proof for the forumula $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$.

2. [**Falkner Section 5 Exercise 6**] Prove that for each $x \in \mathbb{Z}$, 6 divides $x^3 - x$. [HINT: First use induction to handle the case where $x \in \mathbb{N}$.]