

HOMEWORK 11
MATH 3345 – SPRING 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 5 Exercise 5]** Prove by induction that for each $n \in \mathbb{N}$, 5 divides $7^n - 2^n$.
2. (a) Let n be an integer with $n > 1$. Prove that n is prime if and only if for every prime p such that $p^2 \leq n$, p does not divide n .
(b) Use part (a) to prove that 29 is prime.
(c) Use part (a) to prove that 101 is prime.
3. **[The sieve of Eratosthenes]**
 - (a) Write all of the integers from 1 to 100 (perhaps as a 10×10 array), and do the following.
 - i. Cross out the number 1.
 - ii. Circle the number 2, then cross out all other numbers divisible by 2.
 - iii. Circle the number 3, then cross out all other numbers divisible by 3.
 - iv. Repeat this procedure: Circle the smallest number d which is not crossed out, then cross out all other numbers divisible by d . Stop once every integer from 1 to 100 is either circled or crossed out.
 - (b) Explain why the circled numbers are all of the primes less than 100.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Explain why 0 is the only integer with infinitely many divisors.
2. **[Falkner Section 4 Exercise 13]** Let $a, b, c \in \mathbb{Z}$. Prove the following statements.
 - (a) If $a|b$ and $a|c$, then $a|(b + c)$ and $a|(b - c)$.
 - (b) If $a|b$ or $a|c$, then $a|bc$.
 - (c) If $a|b$, then $a|(-b)$.
 - (d) If $a|b$, then $(-a)|b$.