

HOMEWORK 12
MATH 3345 – SPRING 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 4 Exercise 8]** Let x and y be rational numbers. Prove the following statements.
 - (a) $-y$ is a rational number.
 - (b) $x - y$ is a rational number.
 - (c) xy is a rational number.
 - (d) If $y \neq 0$, then $1/y$ is a rational number.
 - (e) If $y \neq 0$, then x/y is a rational number.
2. Let $a, b \in \mathbb{N}$. We say that a positive integer $m \in \mathbb{N}$ is a **common multiple** of a and b if $a|m$ and $b|m$.
 - (a) Show that for any $a, b \in \mathbb{N}$, ab is a common multiple of a and b .
 - (b) Prove that for any $a, b \in \mathbb{N}$, there exists a common multiple ℓ of a and b such that $\ell \leq m$ if m is any common multiple of a and b . This number ℓ is called the **least common multiple** of a and b . We write $\ell = \text{lcm}(a, b)$.
 - (c) Give an example of positive integers $a, b \in \mathbb{N}$ such that $\text{lcm}(a, b) = ab$.
 - (d) Give an example of positive integers $a, b \in \mathbb{N}$ such that $\text{lcm}(a, b) < ab$.
 - (e) Explain why there do not exist positive integers a and b such that $\text{lcm}(a, b) > ab$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Let n be an integer. Prove that if $3|n^2$, then $3|n$.
2. Find an integer n such that $4|n^2$ but $4 \nmid n$.