Homework 13
Math 3345 – Spring 2022 – Kutler

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. [**Falkner Section 4 Exercise 10**] Let x be a rational number and let y be an irrational number. Prove the following statements.

   (a) $-y$ is irrational.

   (b) $x - y$ is irrational.

   (c) $y - x$ is irrational.

   (d) If $x \neq 0$, then xy is irrational. [Be sure to explain where you use the condition that $x \neq 0$ in your proof.]

   (e) Is it possible that there is a different proof for part (d) that does not use the condition that $x \neq 0$ but still leads to the conclusion that xy is irrational? Explain your answer.

   (f) $1/y$ is irrational. [You should explain why $y \neq 0$ **must** be true.]

   (g) If $x \neq 0$, then x/y is irrational.

   (h) If $x \neq 0$, then y/x is irrational.

2. [**Falkner Section 8 Exercise 9**] Let $a, b \in \mathbb{R}$. Suppose $a \geq 0$ and $b \geq 0$. Prove that:

   (a) If $a < b$, then $a^2 < b^2$. [Use basic properties of inequalities.]

   (b) If $a^2 \leq b^2$, then $a \leq b$. [Do not use square roots. Use part (a).]

   (c) If $a^2 < b^2$, then $a < b$. [Again, do not use square roots. Use part (b).]

   (d) If $a < b$, then $\sqrt{a} < \sqrt{b}$. [Use part (c).]

3. [**Falkner Section 8 Exercise 11**] Let $a, b \in \mathbb{R}$. Suppose $a > 0$ and $b > 0$. Prove that $\sqrt{a+b} < \sqrt{a} + \sqrt{b}$. [HINT: Use part (c) of the previous exercise.]

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. [**Falkner Section 4 Exercise 12**] Show that for each real number x, $\pi + x$ is irrational or $\pi + x$ is rational.

2. [**Falkner Section 8 Exercise 10**] Let $a, b \in \mathbb{R}$. Prove that if $0 < a < b$, then $a < \sqrt{ab} < b$.