

HOMEWORK 14
MATH 3345 – SPRING 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. Let $a, b, q, r \in \mathbb{Z}$ such that $a = bq + r$.
 - (a) Let $d \in \mathbb{N}$. Prove that d is a common divisor of a and b if and only if d is a common divisor of b and r .
 - (b) Use part (a) to conclude that $\gcd(a, b) = \gcd(b, r)$.
2. Use the Euclidean algorithm to compute the following.
 - (a) $\gcd(36, 22)$
 - (b) $\gcd(96, 112)$
 - (c) $\gcd(162, 31)$
 - (d) $\gcd(-15, 45)$

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. [**Falkner Section 4 Exercise 16**] Let $n \in \mathbb{N}$. Prove that there exists a prime number q such that $n < q \leq 1 + n!$. [HINT: Take q to be any prime which divides $1 + n!$. (How do we know such a prime exists?) Now explain why $q \leq 1 + n!$ and $q > n$ must both be true.]
2. Let $a, b \in \mathbb{N}$. Prove that $\gcd(a, b) \cdot \text{lcm}(a, b) = ab$.