Homework 15

Math 3345 – Spring 2022 – Kutler

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. (a) Use the Euclidean algorithm to compute $\gcd(350, 168)$.
   (b) Find integers x_1 and y_1 such that $350x_1 + 168y_1 = 14$.
   (c) Find integers x_2 and y_2 such that $350x_2 + 168y_2 = 28$.
   (d) Prove that there do not exist integers x and y such that $350x + 168y = 15$.

2. [**Falkner Section 4 Exercise 25**] Let $m \in \mathbb{N}$. Show that
   (a) For all $a \in \mathbb{Z}$, we have $a \equiv a \mod m$. [**Reflexivity**]
   (b) For all $a, b \in \mathbb{Z}$, if $a \equiv b \mod m$, then $b \equiv a \mod m$. [**Symmetry**]
   (c) For all $a, b, c \in \mathbb{Z}$, if $a \equiv b \mod m$ and $b \equiv c \mod m$, then $a \equiv c \mod m$. [**Transitivity**]

3. [**Falkner Section 4 Exercise 26 – modified**] Let $m \in \mathbb{N}$ and $a, b, c, d \in \mathbb{Z}$. Suppose that $a \equiv b \mod m$ and $c \equiv d \mod m$.
   (a) Prove that $a + c \equiv b + d \mod m$.
   (b) Prove that $a - c \equiv b - d \mod m$.
   (c) Prove that $ac \equiv bd \mod m$. [HINT: Since $a \equiv b \mod m$, m divides $b - a$, so $b - a = mk$ for some integer k. Rewrite this as $b = a + mk$. Similarly, $d = c + m\ell$ for some integer ℓ.]

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Without using a calculator, find the natural number k such that $0 \leq k \leq 14$ and k satisfies the given congruence.
   (a) $2^{75} \equiv k \pmod{15}$
   (b) $6^{41} \equiv k \pmod{15}$
   (c) $140^{874} \equiv k \pmod{15}$