Homework 20

Math 3345 – Spring 2022 – Kutler

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. [**Falkner Section 10 Exercise 19 – modified**] Let A, B, and X be sets.

   (a) Prove that if $A \subseteq B$, then $X \setminus B \subseteq X \setminus A$.

   (b) Prove that $A \subseteq X$ if and only if $A = X \setminus (X \setminus A)$. [HINT: Use the result of Homework 19 Exercise 3 to express $X \setminus (X \setminus A)$ in a simpler form.]

   (c) Suppose $A \subseteq X$. Prove that if $X \setminus B \subseteq X \setminus A$, then $A \subseteq B$.

   (d) Show, by giving an example, that the implication
   $$\text{if } X \setminus B \subseteq X \setminus A, \text{ then } A \subseteq B$$
   may be **false** if $A \not\subseteq X$.
   That is, give an example of sets A, B, and X such that $X \setminus B \subseteq X \setminus A$ and $A \not\subseteq B$.

2. [**Falkner Section 10 Exercise 24**] Prove Proposition 10.34(b): Let $\mathscr{A}$ be a nonempty set of sets and let X be any object. Then
   $$x \notin \left(\bigcap_{A \in \mathscr{A}} A \right) \text{ if and only if there exists } A \in \mathscr{A} \text{ such that } x \notin A.$$

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. [**Falkner Section 10 Exercise 26**] Prove Theorem 10.36(b): Let S be a set and let $\mathscr{A}$ be a nonempty set of sets. Then
   $$S \cup \left(\bigcap_{A \in \mathscr{A}} A \right) = \bigcap_{A \in \mathscr{A}} (S \cup A).$$

2. [**Falkner Section 10 Exercise 27**] Let A be a set and let $\mathscr{B}$ be a nonempty set of sets. Show that:

   (a) $A \cup \left(\bigcup_{B \in \mathscr{B}} B \right) = \bigcup_{B \in \mathscr{A}} (A \cup B)$

   (b) $A \cap \left(\bigcap_{B \in \mathscr{B}} B \right) = \bigcap_{B \in \mathscr{A}} (A \cap B)$