

HOMEWORK 21
MATH 3345 – SPRING 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 10 Exercise 33(a)–(d)]** Let A , B , C , and D be sets.
 - (a) Prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$
 - (b) Prove that $(A \cup B) \times C = (A \times C) \cup (B \times C)$ and $A \times (B \cup C) = (A \times B) \cup (A \times C)$.
 - (c) Prove that $(C \times D) \setminus (A \times B) = E \cup F$, where $E = (C \setminus A) \times D$ and $F = C \times (D \setminus B)$.
 - (d) In the special case where $A = [1, 3] = B$ and $C = [2, 4] = D$, draw a picture to illustrate the result you proved for the general case in part (c).

2. **[Falkner Section 10 Exercise 34 – modified]**
 - (a) Let A be a set. Prove that $A \times \emptyset = \emptyset$.
 - (b) Let A and B be sets. Deduce that $A \times \emptyset = B \times \emptyset$.
 - (c) Let A , B , and C be sets, and suppose that $C \neq \emptyset$. Prove that if $A \times C = B \times C$, then $A = B$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 10 Exercise 28]** Find $\mathcal{P}(\{1, 2, 3\})$.
2. **[Falkner Section 10 Exercise 32]** Sketch the rectangle $[1, 4] \times [2, 3]$ in the coordinate plane. (Shade the set of points that belong to this rectangle.)
3. **[Falkner Section 10 Exercise 36]** Let A , B , and C be sets. Prove that

$$A \setminus C \subseteq (A \setminus B) \cup (B \setminus C).$$