HOMEWORK 22
MATH 3345 – SPRING 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. [**Falkner Section 11 Exercise 6**] Let $f(x) = x^2 + 1$ for all $x \in \mathbb{R}$, let $g(y) = \sqrt{y-1}$ for all $y \in [1, \infty)$, and let $h(u) = 1 - u$ for all $u \in [2, 3)$. Find the range of f, the range of g, and the range of h.

2. [**Falkner Section 11 Exercise 15**] Let S and T be sets. Define a function
$$f \colon \mathscr{P}(S) \times \mathscr{P}(T) \to \mathscr{P}(S \cup T)$$
by $f(A, B) = A \cup B$ for all $A \subseteq S$ and $B \subseteq T$.

   (a) Show that f is a surjection.
   (b) Show that f is an injection if and only if S and T are disjoint (i.e., $S \cap T = \varnothing$).

3. [**Falkner Section 11 Exercise 17 – modified**] Let
$$f \colon [1, \infty) \to \mathbb{R}$$
$$x \mapsto x - 1.$$

   (a) Show that $\operatorname{Rng}(f) \subseteq [0, \infty)$. That is, $f(x) \in [0, \infty)$ for every $x \in [1, \infty)$.
   (b) Prove that $\operatorname{Rng}(f) = [0, \infty)$. [HINT: In light of part (a), you need only prove the other inclusion, $[0, \infty) \subseteq \operatorname{Rng}(f)$. That is, for each $y \in [0, \infty)$, you must find some $x \in \operatorname{Dom}(f) = [1, \infty)$ such that $f(x) = y$.]
   (c) Prove that f is an injection.
   (d) Conclude that f is a bijection from $[1, \infty)$ to $[0, \infty)$, and give a formula for the inverse function $f^{-1} \colon [0, \infty) \to [1, \infty)$.
   (e) Sketch the graph of f.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. [**Falkner Section 10 Exercise 35**] Let A, B, C, and D be sets. Suppose that $A \times B = C \times D \neq \varnothing$. Prove that $A = C$ and $B = D$.