Homework 3

Math 3345 – Spring 2022 – Kutler

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. Write **both** the contrapositive and the converse of each conditional sentence below.

   (a) If it is raining, then the ground is wet.

   (b) If $a = 4$, then $a^2 = 16$.

   (c) If $a \neq b$, then $a^4 \neq b^4$.

   Do not worry about the truth value of any of these statements.

2. [**Falkner Section 2 Exercise 6**] Let P and Q be logical sentences. Use a truth table to prove that $\neg(P \Rightarrow Q)$ is logically equivalent to $P \wedge \neg Q$.

3. [**Falkner Section 2 Exercise 7**] Let x and y be real numbers.

   (a) Let A be the sentence "If $x + y > 0$, then $x > 0$ or $y > 0$." Use Theorem 2.10 and one of De Morgan's laws to show that $\neg A$ is logically equivalent to "$x + y > 0$ and $x \leq 0$ and $y \leq 0$." Be careful not to skip any steps.

   (b) Is the sentence A in part (a) true, or is $\neg A$ true? Explain why.

   (c) Let B be the sentence "If $x + y > 2$, then $x > 2$ or $y > 2$." Is B true, or is $\neg B$ true, or is it impossible to say without further information about the specific values of x and y? (Hint: Can you find specific values for x and y for which B is true? If so, give an example of such values. Can you find other specific values for x and y for which $\neg B$ is true? If so, give an example of such values.)

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. Use De Morgan's laws to find a sentence which is logically equivalent to

$$\neg\Big[\big((x > 1) \wedge (x < 3)\big) \vee \big((x \geq 4) \wedge (x < 7)\big)\Big]$$

   and which does not use the logical connective "$\neg$".

2. Is $(P \Rightarrow Q) \Rightarrow R$ logically equivalent to $P \Rightarrow (Q \Rightarrow R)$? Use a truth table to justify your answer.