Homework 4

Math 3345 – Spring 2022 – Kutler

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 2 Exercise 9]** Let $P \operatorname{xor} Q$ mean "P exclusive or Q." In other words, $P \operatorname{xor} Q$ should be true just when **exactly one** of P or Q is true.
   (a) Write out the truth table for $P \operatorname{xor} Q$.
   (b) Show by a truth table that $P \operatorname{xor} Q$ is logically equivalent to $(P \wedge \neg Q) \vee (Q \wedge \neg P)$.
   (c) Show by truth tables that the following four sentences are logically equivalent:
   $$P \operatorname{xor} Q, \qquad \neg(P \Leftrightarrow Q), \qquad (\neg P) \Leftrightarrow Q, \qquad P \Leftrightarrow (\neg Q).$$
   (d) Show by a truth table that $(\neg P) \Leftrightarrow (\neg Q)$ is logically equivalent to $P \Leftrightarrow Q$.

2. **[Falkner Section 2 Exercise 14]** Show by means of an explanation in words that the sentence $(P \wedge Q) \Rightarrow (P \vee Q)$ is a tautology. (As usual, you should use the method of conditional proof.)

3. **[Falkner Section 2 Exercise 15]** Use the method of conditional proof to explain in words why the sentence
   $$\big\{(P \vee Q) \wedge [(P \Rightarrow R) \wedge (Q \Rightarrow S)]\big\} \Rightarrow (R \vee S)$$
   is a tautology. **Use the method of conditional proof. You do NOT need to use the book's method of "discharging assumptions."**

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. (a) **[Falkner Section 2 Exercise 4]** Suppose that $P \vee Q$ is true and $\neg Q$ is true. Explain why it follows that P must be true.
   (b) Prove that the conditional sentence
   $$\big[(P \vee Q) \wedge \neg Q\big] \Rightarrow P$$
   is a tautology (that is, it is true for all possible truth values of P and Q).
   Do not use a truth table. Rather, use your work from part (a) to write a conditional proof.

2. **[Falkner Section 2 Exercise 13]** Show by means of an explanation in words that the sentence $P \Rightarrow (P \vee Q)$ is a tautology.