

HOMEWORK 5
MATH 3345 – SPRING 2022 – KUTLER

Please complete the following problems on your own paper. Solutions should be written clearly, legibly, and with appropriate style.

1. **[Falkner Section 2 Exercise 17]** Use the method of conditional proof to explain why the sentence

$$(P \Rightarrow Q) \Rightarrow \{[P \Rightarrow (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)\}$$

is a tautology. **You do NOT need to use the book’s method of “discharging assumptions.” However, please be clear about which assumptions you are making, and why you may make these assumptions.**

2. **[Falkner Section 3 Exercise 1]** For each of the following sentences, write out what it means in words, state whether it is true or false, and prove your statement.

- (a) $(\exists x \in \mathbb{R})(2x + 7 = 3)$.
- (b) $(\forall x \in \mathbb{R})(2x + 7 = 3)$.
- (c) $(\exists x > 0)(2x + 7 = 3)$.
- (d) $(\forall x > 0)(2x + 7 = 3)$.
- (e) $(\exists x \in \mathbb{R})(x^2 - 4x + 3 > 0)$.
- (f) $(\forall x \in \mathbb{R})(x^2 - 4x + 3 > 0)$.
- (g) $(\exists x \geq 7)(x^2 - 4x + 3 > 0)$.
- (h) $(\forall x \geq 7)(x^2 - 4x + 3 > 0)$.
- (i) $(\forall x \in \mathbb{R})(x^2 - 2x + 2 > 0)$.
- (j) $(\forall x \geq 0)(\sqrt{x+3} = \sqrt{x} + \sqrt{3})$.
- (k) $(\exists x \geq 0)(\sqrt{x+3} = \sqrt{x} + \sqrt{3})$.

Practice Problems

It is strongly recommended that you complete the following problems. There is no need to write up polished, final versions of your solutions (although you may find this a useful exercise). Please do not submit any work for these problems.

1. **[Falkner Section 2 Exercise 22]** Let A be the sentence $(P \Rightarrow Q) \Rightarrow \{[P \Rightarrow (Q \Rightarrow R)] \Rightarrow (P \Rightarrow R)\}$. We saw in Exercise 17 that A is a tautology. Let B be the converse of A . Write out B in terms of P , Q , and R . Then show that B is not a tautology, by finding a combination of truth values for P , Q , and R that makes B false. You should be able to do this without writing out a truth table.